

DECOMPOSITION RATE WITH RESPECT TO TIME IN DAYS

The decomposition rate and the decomposition time of the organic matter in the soil are controlled by many factors, such as physical and chemical properties of organic matter itself, soil characteristics, climate and soil organisms. In the farmland ecosystem, straw is decomposed and transformed through the interaction of soil microorganisms and soil animals, so the soil organisms have an irreplaceable role in the decomposition of organic matter such as straw!

We will follow the decomposition model of litter (process through which dead organic material is broken down into particles of progressively smaller size, until the structure can no longer be recognized, and organic molecules are mineralized to their prime constituents: H_2O , CO_2 and mineral components)

Decomposition of litter goes through at least two processes namely nutrient control stage and the cellulose control stage. The litter decomposition is faster in the first stage and slower in the second stage. Now, there are two models to estimate the decomposition rate of litter - Namely, Exponential decay and the Linear decay. Periodic change in the litter decomposition rate may be the reason for the above two different change models.

EXPONENTIAL DECAY:

For the exponential decay, let us take the simplest model equation:

$$\frac{x_t}{x_0} = e^{-k t} \quad (1)$$

Where, x_0 is the initial dry matter mass of the litter (g), x_t is the residual dry matter mass after a period of decomposition time 't'. and k is the average decomposition rate of farmland organic waste established the ecological process and the regulation mechanism of the decomposition and the transformation of organic matter in the farmland ecosystem and provided a new model for the simulation research of soil agricultural system and the efficient utilization of organic waste resources. At the same time, it also provides a basis for establishing a typical farmland ecosystem decomposer sub-system model.

METHOD THAT WE WILL USE TO STUDY THE DECOMPOSITION RATES:

THE LITTER BAG METHOD

1. We will cut the air - dried wheat straw into 3 - 5 cm sections, dried at 65°C for 8 hours.
2. Weighted 10 grams of wheat straw and put them in 10 cm × 10 cm mesh bags.
3. Then we will prepare the soil with our engineered bacteria in it.
4. Two different soils we will prepare as: Control area that contains our engineered bacteria and Normal soil without our engineered bacteria.
5. We will insert the mesh bags containing the straws into these two different soils and measure the decomposition rate.

Environmental conditions: Water will be regularly sprayed to keep the soil moist and provide the optimum temperature for the bacterial consortium to grow.

DATA TO BE MEASURED:

Before Performing the experiment:

1. Initial elemental analysis of the soil to get the N, P, K, and other ions concentration in the soil.
2. Moisture content of the soil.
3. Water Percolation rate of the soil.

During performing the experiment (Every day or two):

1. Physical characteristics of the straw.
2. Weight of the straw decomposed by weighing it at regular intervals.
3. Ions concentration of N, P, K, and other ions during the experiment.
4. Water Percolation rate, is it increased or decreased?

Every week, measure the weight of the system (Soil + Mesh Bag). Decrease in weight will give the analysis of gases that escaped to the surroundings.

ESTABLISHMENT OF THE MODEL:

Let (x_i, y_i) , $(i = 1, 2, 3, \dots, n)$ be the experimental data and (x_k, y_k) to conform the experimental model:

$$y = a(1 - e^{-kx}) \tag{2}$$

Where, a and k are the parameters to be determined. In order to determine the estimated value of 'k' we will use the non-linear least squares criterion and the residual sum of squares [Q (a,k)] is calculated using the formula:

$$Q(a, k) = \sum_{i=1}^n [y_i - a(1 - e^{-kx_i})]^2 \tag{3}$$

The algorithm used to reach the minimum as target optimization parameter includes: Gauss - Newton Method, Marquardt optimization method and DUD method.

The value of a and k are to be determined experimentally, Unfortunately, we don't have initial experimental data to determine the value of a and k respectively however, using literature Search, the value of 'a' and 'k' can be determined:

Treatments	Parameter		Regression Squares sum	Residual Squares Sum	Index of Correlation
	a	k			
Original Soil	116.17	0.0096	14504.78	88.80	0.993915

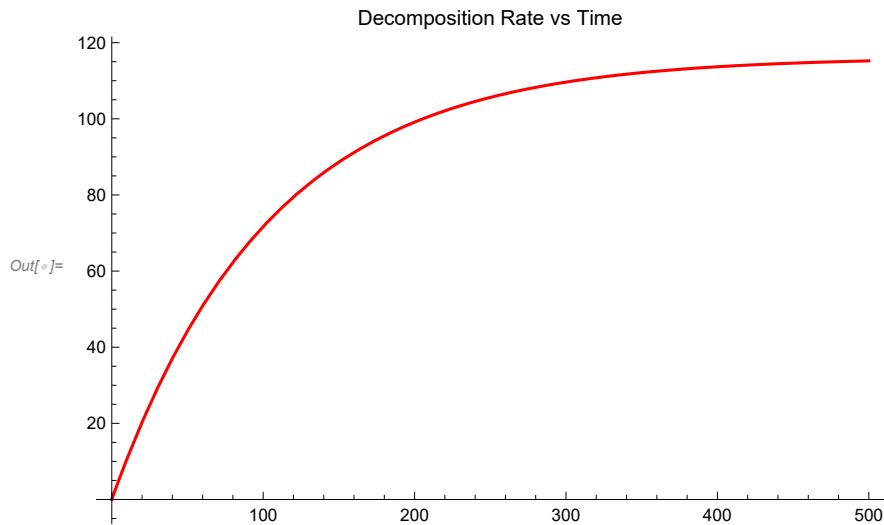
Table is taken from the literature 01

On Plotting the above data and simulating we get:

In[]:= $y = 116.17(1 - E^{-0.0096 x})$

Out[]:= $116.17(1 - e^{-0.0096 x})$

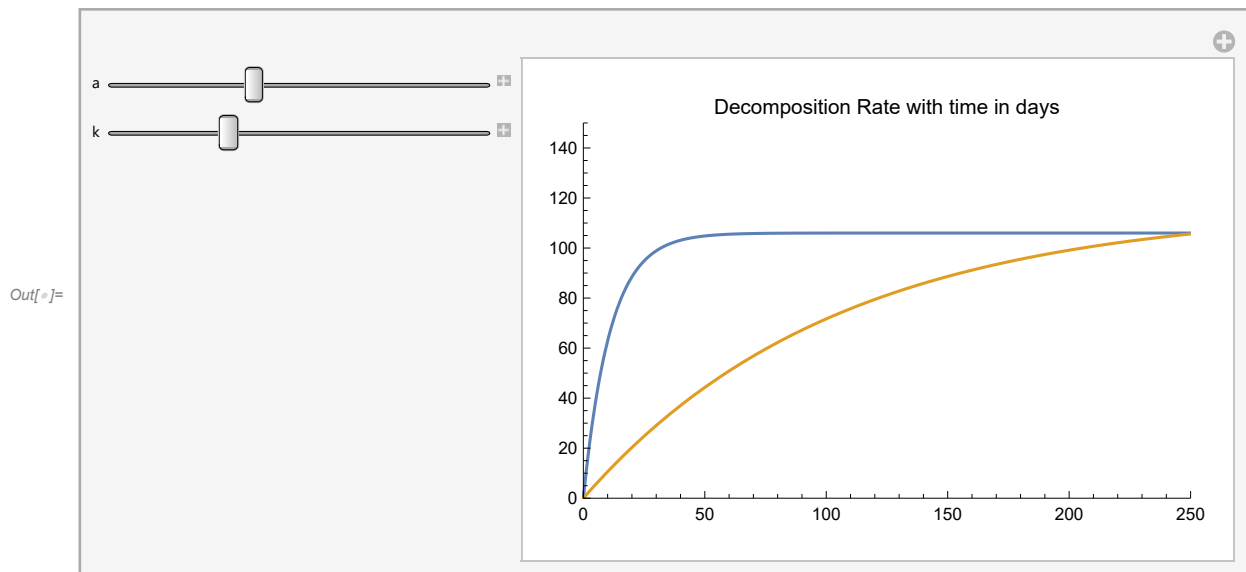
```
In[*]:= Plot[y, {x, 0, 500}, PlotStyle -> Red, PlotLabel -> "Decomposition Rate vs Time"]
```



```
In[*]:= z = a (1 - E-k x)
```

```
Out[*]= a (1 - e-k x)
```

```
In[*]:= Manipulate[Plot[{a (1 - E-k x), 116.17 (1 - E-0.0096 x)}, {x, 0, 250}, PlotRange -> {{0, 250}, {0, 150}}, PlotLabel -> "Decomposition Rate with time in days"], {a, 80, 150, 1}, {k, 0, 0.3, 0.001}]
```



Therefore, Equation 2 can be used to describe the wheat straw decomposition in the fields. From the above manipulation plot, we get the value of 'k' as 0.09 and value of 'a' as 106 respectively which must be targeted through wet lab experiments.

CALCULATION OF HUMIDITY AND TEMPERATURE CONTROL

Specific humidity (or moisture content) is the ratio of the mass of water vapor to the total mass of the air parcel. Now, Mathematically, Specific Humidity or Moisture content in the

air is given by:

$$\text{Moisture Content}_{\text{air}} = \left(6.11 \times 10^{\frac{7.5 \times \text{Dew Point}}{237.3 + \text{Dew Point}}} \right) \quad (1)$$

where, dew point is the temperature to which air must be cooled to become saturated with water vapor, assuming constant air pressure and water content. When cooled below the dew point, moisture capacity is reduced and airborne water vapor will condense to form liquid water known as dew. Mathematically, Dew Point is given by:

$$\text{Dew Point} = T_d = T - \frac{(100 - \text{Relative Humidity})}{5} \quad (2)$$

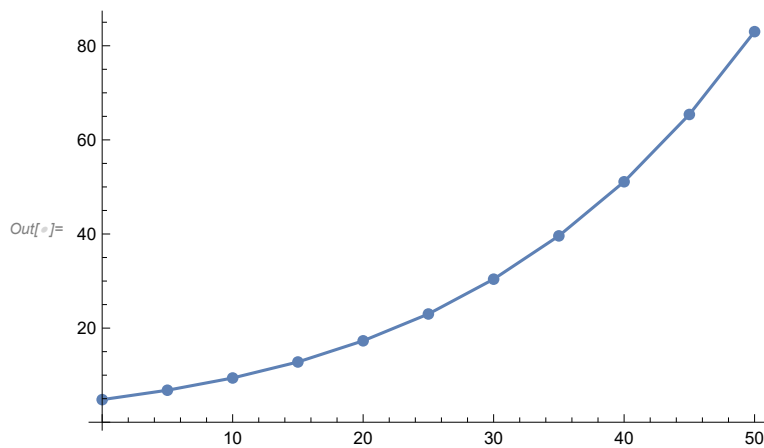
Now, to get the relation between Dew Point and Relative humidity We have,

```
In[*]:= data = {{0, 4.8}, {5, 6.8}, {10, 9.4}, {15, 12.8}, {20, 17.3},
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           {25, 23}, {30, 30.4}, {35, 39.6}, {40, 51.1}, {45, 65.4}, {50, 83}}
```

```
Out[*]:= {{0, 4.8}, {5, 6.8}, {10, 9.4}, {15, 12.8}, {20, 17.3}, {25, 23}, {30, 30.4}, {35, 39.6}, {40, 51.1}, {45, 65.4}, {50, 83}}
```

```
In[*]:= plot1 = ListPlot[data, Joined → True, PlotMarkers → Automatic]
```



```
In[*]:= f[T_] := a T3 + b T2 + c T + d
```

```
In[*]:= z = FindFit[data, f[T], {a, b, c, d}, T]
```

```
Out[*]:= {a → 0.000430769, b → 0.00107226, c → 0.432821, d → 4.66993}
```

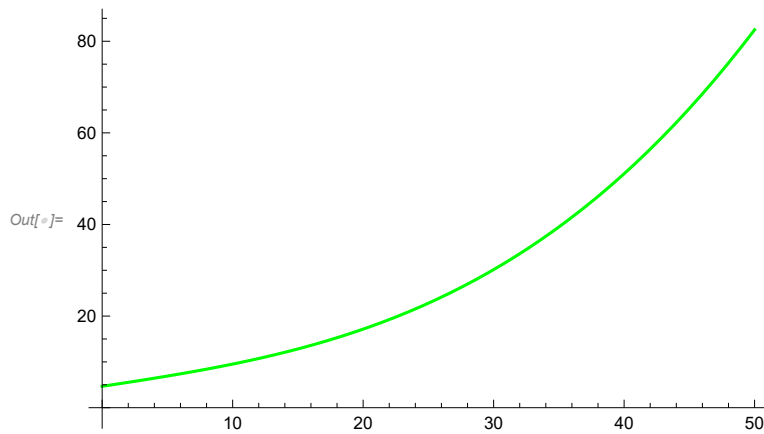
```
In[*]:= g[T_] = f[T] /. z
```

```
Out[*]:= 4.66993 + 0.432821 T + 0.00107226 T2 + 0.000430769 T3
```

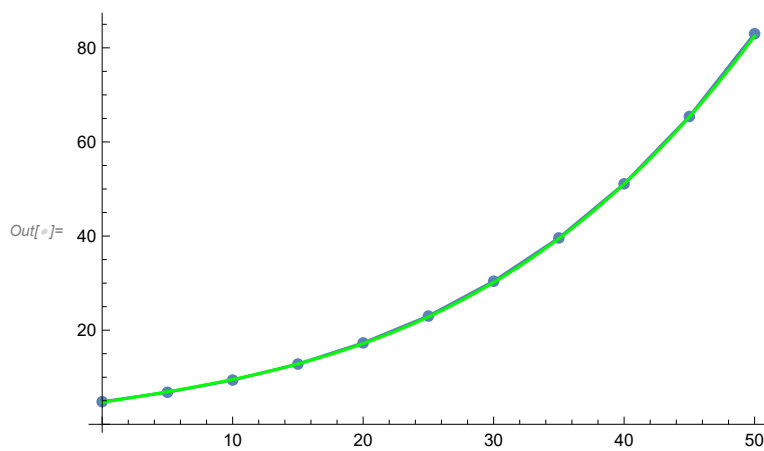
```
In[*]:= g[25]
```

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Out[*]:= 22.8914
```

```
In[ ]:= plot2 = Plot[4.669 + 0.432 T + 0.001 T2 + 0.00043 T3, {T, 0, 50}, PlotStyle -> Green]
```



```
In[ ]:= Show[plot1, plot2]
```



Hence, the equation satisfying the relation between the Dew Point and the Temperature is

Given by: $4.669 + 0.432 T + 0.001 T^2 + 0.00043 T^3$

Placing this Equation in the place of Relative Humidity in Equation 2, we get,

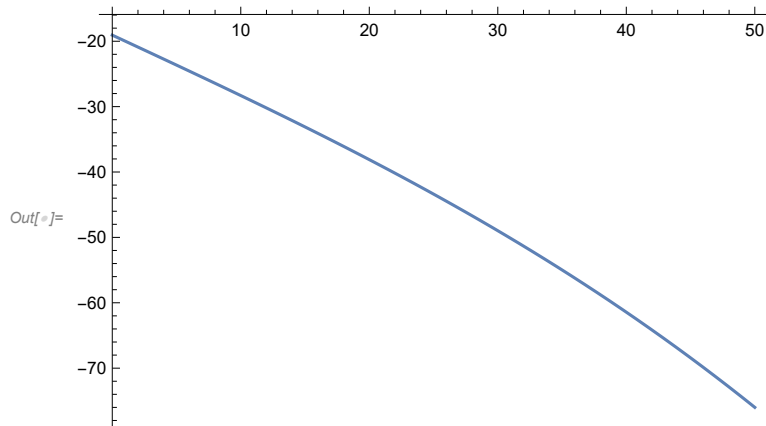
$$\text{Dew Point} = T - \left[\frac{100 - 4.669 + 0.432 T + 0.001 T^2 + 0.00043 T^3}{5} \right],$$

Which on simplifying gives, Dew Point = $-8.6 \times 10^{-5} T^3 - 0.0002 T^2 - 0.9136 T - 19.0662$

```
In[ ]:= dewpoint = (-8.6 * 10^-5) x^3 - 0.0002 x^2 - 0.9136 x - 19.0662
```

```
Out[ ]:= -19.0662 - 0.9136 x - 0.0002 x^2 - 0.000086 x^3
```

```
In[ ]:= Plot[dewpoint, {x, 0, 50}]
```



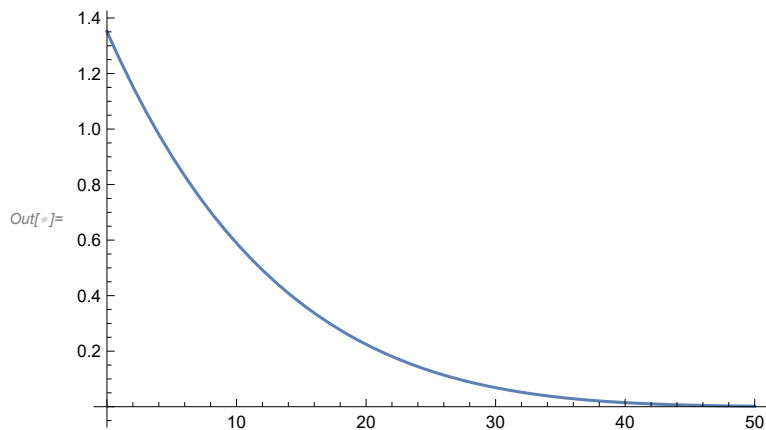
And Hence the Moisture content is given by: Moisture Content =

$$\left(6.11 \times 10^{\frac{7.5(-19.0662-0.9136 T-0.0002 T^2-0.000086 T^3)}{237.3-19.0662-0.9136 T-0.0002 T^2-0.000086 T^3}} \right)$$

```
In[ ]:= moisturecont = 6.11 * 10^{\frac{7.5(-19.0662-0.9136 T-0.0002 T^2-0.000086 T^3)}{237.3-19.0662-0.9136 T-0.0002 T^2-0.000086 T^3}} // FullSimplify
```

```
Out[ ]:= 6.11 \times 10^{\frac{1.66275 \times 10^6 + T(79674.4 + T(17.4419 + 7.5 T))}{-2.5376 \times 10^6 + T(10623.3 + T(2.32558 + 1. T))}}
```

```
In[ ]:= Plot[moisturecont, {T, 0, 50}]
```



Hence, the Moisture content in the air is related to temperature as: $M'(T) =$

$$6.11 \times 10^{\frac{1.66275 \times 10^6 + T(79674.4 + T(17.4419 + 7.5 T))}{-2.5376 \times 10^6 + T(10623.3 + T(2.32558 + 1. T))}}$$

```
In[ ]:= Integrate[6.11 * 10^{\frac{7.5(-19.0662-0.9136 T-0.0002 T^2-0.000086 T^3)}{237.3-19.0662-0.9136 T-0.0002 T^2-0.000086 T^3}}, {T, 0, 100}] // N
```

```
Out[ ]:= 14.8648
```

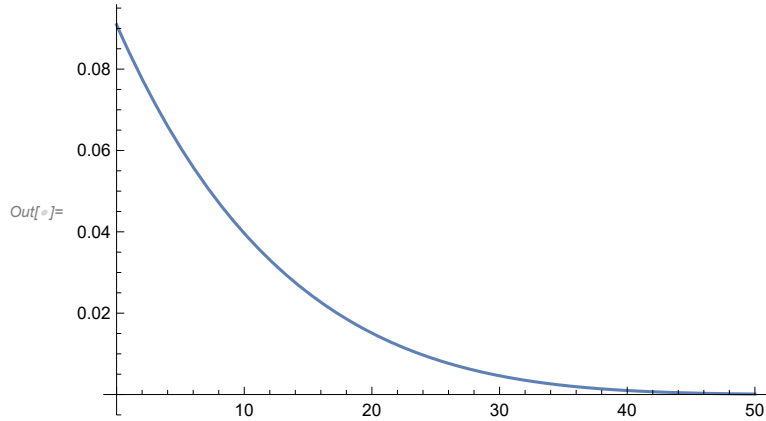
Hence, the normalizing constant is 14.8648, Therefore Normalized Function will be given as:

$$M(T) = 0.4110 \times 10^{\frac{1.66275 \times 10^6 + T(79674.4 + T(17.4419 + 7.5 T))}{-2.5376 \times 10^6 + T(10623.3 + T(2.32558 + 1. T))}}$$

```
In[ ]:= normoistcont = 0.4110 * 10 7.5 (-19.0662-0.9136 T-0.0002 T2-0.000086 T3) / 237.3 -19.0662-0.9136 T-0.0002 T2-0.000086 T3 // FullSimplify
```

```
Out[ ]:= 0.411 × 10 1.66275×106+T (79 674.4+T (17.4419+7.5 T)) / -2.5376×106+T (10 623.3+T (2.32558+1. T))
```

```
In[ ]:= Plot[normoistcont, {T, 0, 50}]
```



Now, To incorporate the effects of fluctuating soil temperatures on decomposition rates in the regression models, the independent variable, time, was transformed. As a first step, only days with a mean soil temperature > 0 degree Celsius were included. A second step was to use the straight temperature sum (T_{sum}).

Temperature sums (Q_{sum}) were also derived from Q_{10} relationship as:

$T_{\text{sum}} = \sum_{i=1}^x \max \left\{ \frac{T_i}{23}, 0 \right\}$ (Assuming the mean soil temperature of India is around 23 degree Celsius)

and

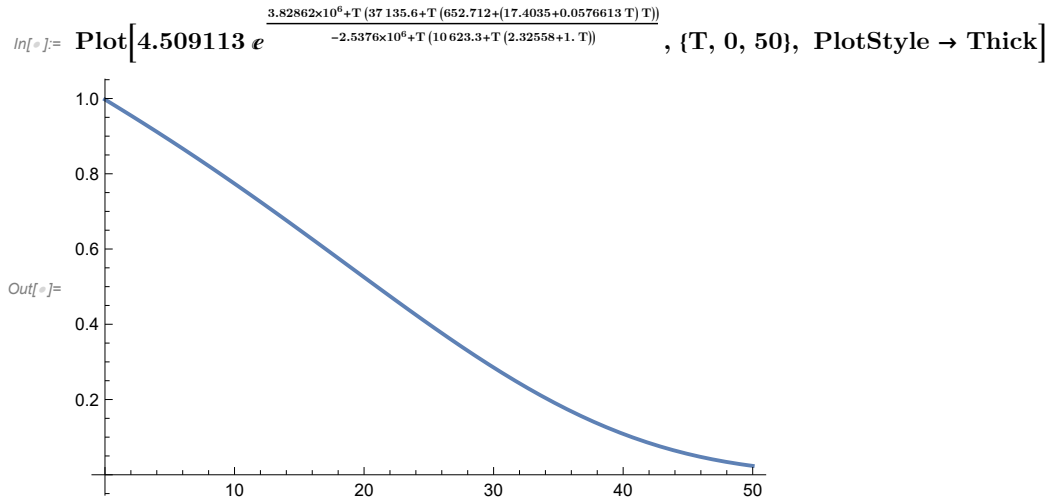
$Q_{\text{sum}} = \sum_{i=1}^x Q_{10}^{\frac{(T_i - 23)}{10}} ; T_i > 0$

So, The combined effect of temperature and moisture was incorporated into the numerical simulations as a control function ranging in value from 0 to 1, which modified rate constants:

$f(T, M(T)) = 1.78^{\frac{(T-23)}{10}} \times 0.4110 \times 10^{\frac{1.66275 \times 10^6 + T (79 674.4 + T (17.4419 + 7.5 T))}{-2.5376 \times 10^6 + T (10 623.3 + T (2.32558 + 1. T))}}$

```
In[ ]:= F[T_] = 1.78 (T-23) / 10 * 0.4110 * 10 7.5 (-19.0662-0.9136 T-0.0002 T2-0.000086 T3) / 237.3 -19.0662-0.9136 T-0.0002 T2-0.000086 T3 // FullSimplify
```

```
Out[ ]:= 4.50911 e 3.82862×106+37 135.6 T+652.712 T2+17.4035 T3+0.0576613 T4 / -2.5376×106+10 623.3 T+2.32558 T2+1. T3
```



Here $F[T]$ will control the Moisture and Soil Temperature fluctuations in the environment whose value varies between 0 and 1. Where, Moisture is controlled by temperature as it is written as a function of temperature to reduce the number of independent variables.

DECOMPOSITION OF LABILE AND REFRACTORY COMPONENTS IN THE WHEAT STRAW

FOLLOWING THE TWO COMPARTMENT MODEL AND IGNORING THE MOISTURE AND TEMPERATURE FLUCTUATIONS:

Let us assume that the total mass is composed of Labile Component (M_L) and Refractory Component (M_R). Equivalently, $M = M_L + M_R$.

Following first order Differential Equations, we have,

$$\text{For Labile Component: } \frac{dM_L}{dt} = -K_L M_L \quad (1)$$

$$\text{For Refractory Component: } \frac{dM_R}{dt} = -K_R M_L \quad (2)$$

Normalized parallel first order differential follows: $\frac{M}{M_0} = \alpha e^{-K_R t} + (1 - \alpha) e^{-K_L t}$

where, $F(T)$ is calculated above as: $F(T) = 4.50911 e^{\frac{3.82862 \times 10^6 + 37135.6 T + 652.712 T^2 + 17.4035 T^3 + 0.0576613 T^4}{-2.5376 \times 10^6 + 10623.3 T + 2.32558 T^2 + 1. T^3}}$

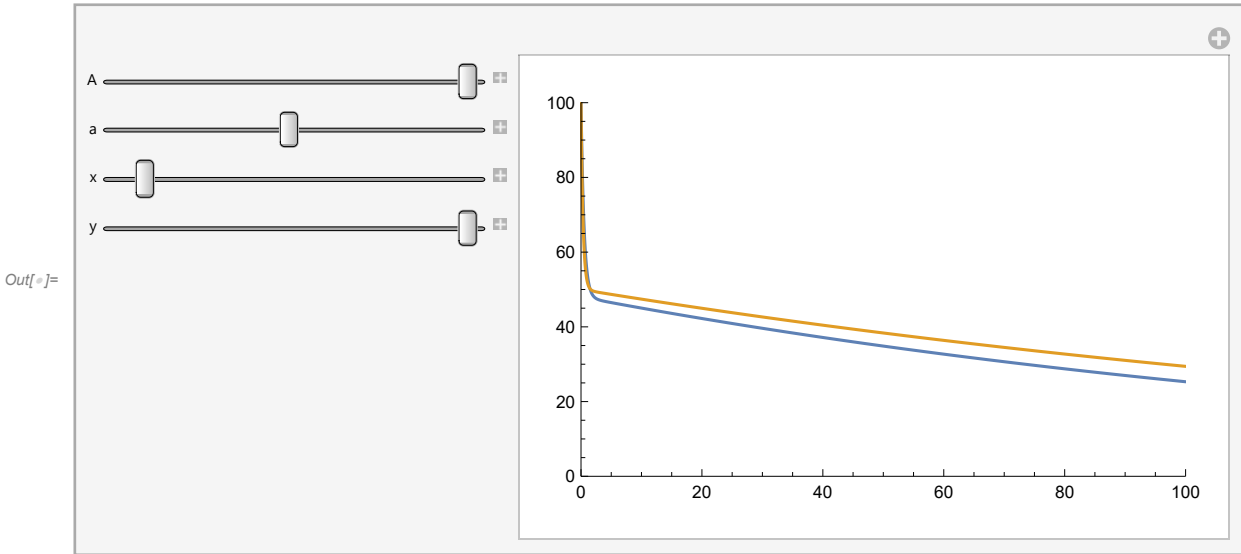
Now, From Literature Search, We have that $K_R = 0.00530$ (Degree-Day) and $K_L = 2.96$ (Degree-Day) so, we can vary K_R and K_L

Let $K_L = y$ and $K_R = x$, $\alpha = a$, A is the initial mass of the straw (in g) at $t = 0$, and t is time in days


```

In[ ]:= Manipulate[Plot[{A * (a (E-(x)(t)) + (1 - a) (E-(y)(t))), (100 * ((0.5) E-0.00530 * t + (1 - 0.5) E-2.96 t))},
{t, 0, 100}, PlotRange -> {{0, 100}, {0, 100}}, {A, 0, 100}, {a, 0, 1}, {x, 0, 0.1}, {y, 0, 2}]

```



Now, Adding the factor of moisture and temperature of the air into our decomposition equation, we get, Overall mass (Labile + Refractory Component both included) decomposition as:

```

In[ ]:= control = DSolve[M'[t] == -k M[t] * 4.509113 e $\frac{7.5 (-19.0662 - 0.9136 T - 0.0002 T^2 - 0.000086 T^3)}{237.3 - 19.0662 - 0.9136 T - 0.0002 T^2 - 0.000086 T^3}$ , M[t], t] // FullSimplify

```

```

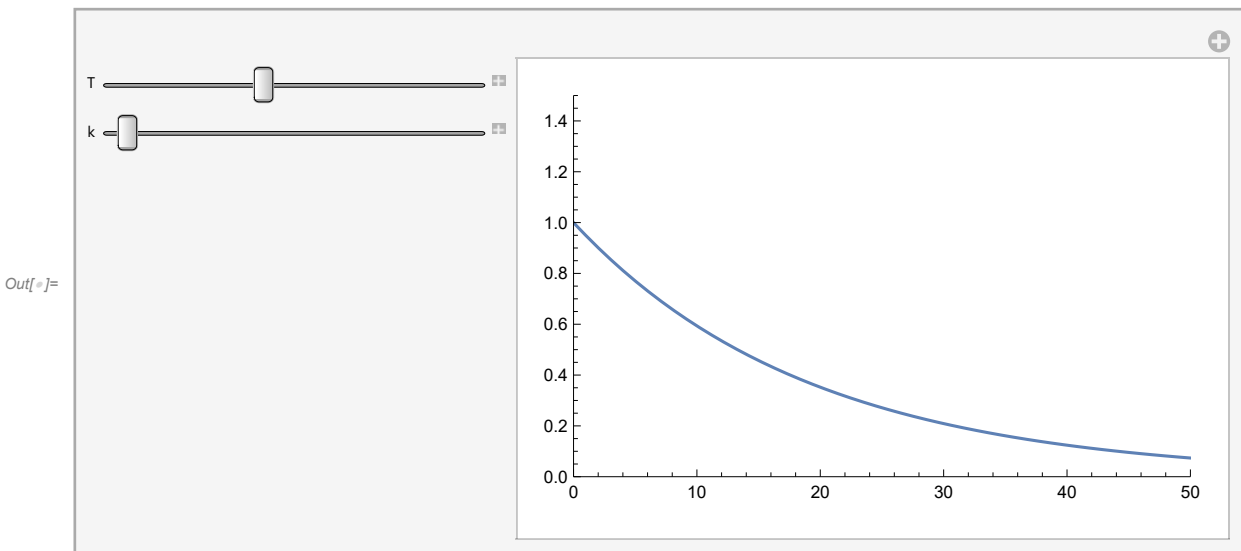
Out[ ]:= {{M[t] -> e $-\frac{1.66275 \times 10^6 + T (79674.4 + T (17.4419 + 7.5 T))}{-2.5376 \times 10^6 + T (10623.3 + T (2.32558 + 1. T))} k t$  c1}}

```

```

In[ ]:= Manipulate[Plot[E $-\frac{1.66275 \times 10^6 + T (79674.4 + T (17.4419 + 7.5 T))}{-2.5376 \times 10^6 + T (10623.3 + T (2.32558 + 1. T))} k t$ , {t, 0, 50}, PlotRange -> {{0, 50}, {0, 1.5}},
{T, 0, 150}, {k, 0, 0.5}]

```



FOUR COMPONENT MODEL:

The model consisted of four mass components: labile (M_L) and refractory (M_R) components of the original straw and active (M_A) and stabilized (M_S) pools composed of decomposition

products. The active pool was assumed to include microbial biomass and non stabilized decomposition products. All transformation will follow the first order kinetics. Product formation and mass loss as CO₂ will be proportional depending on a yield efficiency factor (ϵ). However, only a portion (γ) of the active pool turnover was assumed to become stabilized.

To relate the conceptual components of the model to measured chemical components we assumed that a constant fraction (Φ) of the active pool consisted of water-soluble parts of microbial biomass and metabolites. The initial water-solubles composed the labile fraction and the remaining part of the plant material was included in the refractory component. Thus simulated water-solubles (W) included the labile pool and a fraction of the active pool and insolubles (I) included the refractory and stabilized components and the remaining fraction of the active component. i.e.,

$$W = M_L + \Phi M_A \quad (1)$$

$$I = M_R + M_S + (1 - \Phi) M_A \quad (2)$$

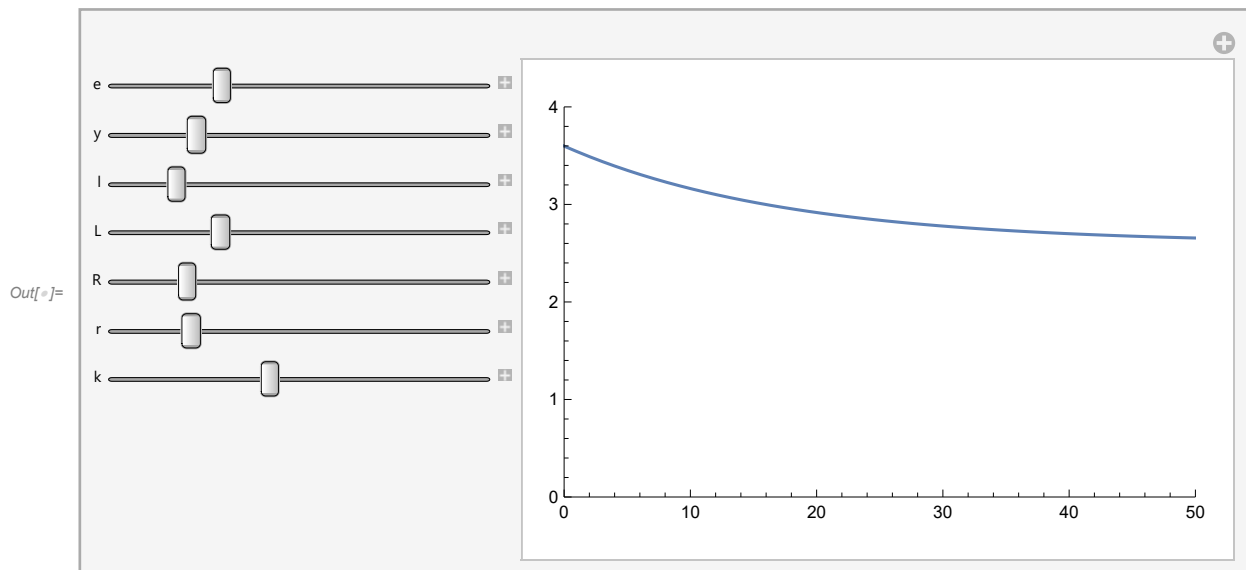
Let us look at the active pool of the straw. The active pool was assumed to include microbial biomass and non stabilized decomposition products. It follows the First Order Kinetics as follows (without considering temperature and moisture effects):

$$\frac{dM[t]}{dt} = [\epsilon \{1 \times L + r \times R + (1 - \gamma) k \times M[t]\} - k \times M[t]]$$

`In[*]:= DSolve[{M'[t] == e (1 * L + r * R + (1 - y) * k * M[t]) - k * M[t]}, M[t], {t}] // FullSimplify`

`Out[*]:= {{M[t] -> \frac{e (1 L + r R)}{k + e k (-1 + y)} + e^{k t (-1 + e - e y)} c_1}}`

`In[*]:= Manipulate[Plot[\frac{e * (1 * L + r * R)}{k + e k (y - 1)} + E^{k t (e - e y - 1)}, {t, 0, 50}, PlotRange -> {{0, 50}, {0, 4}}, {e, 0.1, 1}, {y, 0, 0.999}, {l, 0.1, 0.5}, {L, 0, 10}, {R, 0, 10}, {r, 0, 0.01}, {k, 0.1, 0.05}]`



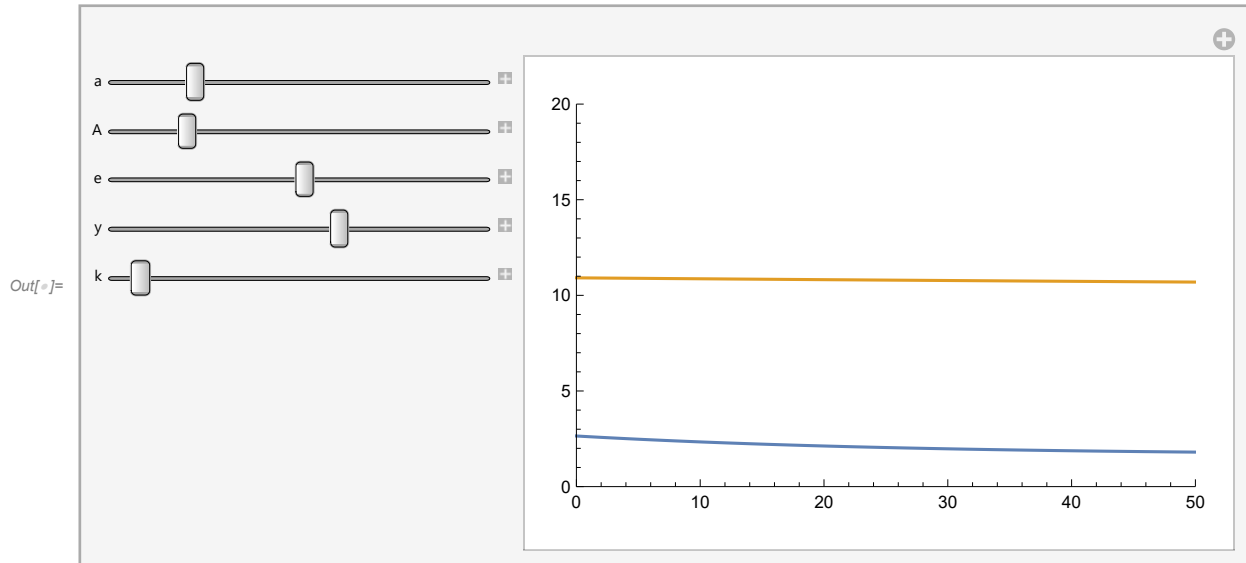
Similarly, let us look at the Stabilized pools of straw, as followed from the first order kinetics as (without considering temperature and moisture effects):

$$\frac{dM[t]}{dt} = (\epsilon\gamma) a \times A - k \times M[t]$$

In[]:= DSolve[{M'[t] == (e * y) a * A - k * M[t]}, M[t], t]

$$\text{Out[]} = \left\{ \left\{ M[t] \rightarrow \frac{a A e y}{k} + e^{-k t} c_1 \right\} \right\}$$

In[]:= Manipulate[Plot[$\left\{ \frac{a * A * e * y}{k} + E^{-k t}, \frac{0.0293 * 10 * 0.36 * 0.47}{0.005} + E^{-0.005 t} \right\}$, {t, 0, 50},
PlotRange -> {{0, 50}, {0, 20}}, {a, 0, 0.5}, {A, 0, 10}, {e, 0.1, 1}, {y, 0.1, 1}, {k, 0.000001, 1}]



We are now in the position to develop the model for change in CO₂ Concentrations using Active pools and Stabilized pools as Follows:

$$\frac{dCO_2}{dt} = (1 - \epsilon) \{ l \times L + r \times R + a \times A + s \times S \}$$

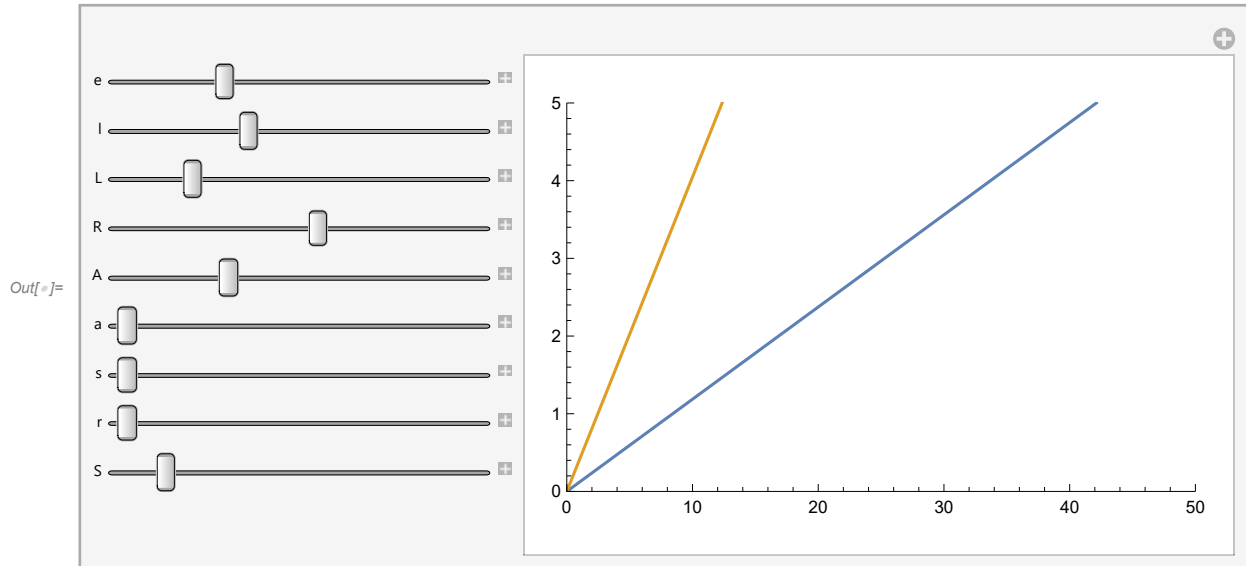
In[]:= DSolve[C'[t] == (1 - e) (l * L + r * R + a * A + s * S), C[t], t] // FullSimplify

$$\text{Out[]} = \{ \{ c_t \rightarrow -((-1 + e) (a A + l L + r R + s S) t) + c_1 \} \}$$

```

In[ ]:= Manipulate[Plot[{{(1 - e) * (l * L + r * R + a * A + s * S) t},
  {(1 - 0.36) (0.367 * 1 + 0.00674 * 4 + 0.0293 * 8 + 0.0005 * 8) t}},
  {t, 0, 50}, PlotRange -> {{0, 50}, {0, 5}}, {e, 0, 1}, {l, 0, 0.5}, {L, 0, 5},
  {R, 0, 5}, {A, 0, 10}, {a, 0, 0.05}, {s, 0, 0.05}, {r, 0, 0.01}, {S, 0, 10}]

```



Total nitrogen in the litter was simulated by assuming constant concentrations in each of the four conceptual litter fractions. In the litter bag analyses, nitrogen contents were not determined separately for each chemical component. The initial nitrogen concentration of water-solubles was estimated assuming the decrease in total nitrogen over the first sampling interval was wholly from the water-soluble component, since only this fraction showed a decrease in mass. As per literature Review, Estimates for the labile (soluble) and refractory (insoluble) nitrogen concentrations were 6 and 0.3%, respectively.